Activity 8

Use the accompanying mark scheme to mark these questions from January 2019 paper 1

1. Find

$$\int\left(\frac{2}{3}x^3 - \frac{1}{2x^3} + 5\right)\mathrm{d}x$$

simplifying your answer.

(4)

Question Number	Scheme	Marks
1.	$\int \frac{2}{3}x^3 - \frac{1}{2x^3} + 5\,\mathrm{d}x = \frac{2}{3}\times\frac{x^4}{4} - \frac{1}{2}\times\frac{x^{-2}}{-2} + 5x + c$	M1 A1
	$= \frac{1}{6}x^4 + \frac{1}{4}x^{-2} + 5x + c$	A1 A1
		(4 marks)

M1 For raising any power by 1 eg. $x^3 \to x^4$, $x^{-3} \to x^{-2}$, $5 \to 5x$ or eg. $x^3 \to x^{3+1}$

A1 For two of $\frac{2}{3}\times\frac{x^4}{4}$, $-\frac{1}{2}\times\frac{x^{-2}}{-2}$, $+5x$ correct (un-simplified) . Accept $5x^1$

This may be implied by a correct simplified answer

A1 For two of $\frac{1}{6}x^4$, $+\frac{1}{4}x^{-2}$, $+5x$ correct and in simplest form. Accept forms such as $\frac{x^4}{6}$, $\frac{1}{4x^2}$,

CONDONE $+\frac{0.25}{x^2}$ but NOT $\frac{1/4}{x^2}$, $\frac{5x}{1}$, $-\left(-\frac{1}{4}x^{-2}\right)$

A1 Fully correct and simplified with $+c$ all on one line. Accept simplified equivalents (see above) and ignore any spurious notation. ISW after a correct simplified answer is achieved.

A

$$\frac{dy}{dx} = \frac{2}{3}x^3 - \frac{1}{2x^3} + 5$$

$$= \cancel{\frac{2x^4}{3 \cdot 4} + \frac{2x^4}{4} + \frac{5x^1}{1}}$$

$$= \frac{2}{3}x^3 - 2x^{-3} + 5$$

$$y = \frac{\frac{2}{3}x^4}{4} - \frac{2x^{-2}}{-2} + \frac{5x^1}{1}$$

$$y = \frac{1}{6}x^4 - 1x^{-2} + 5x$$

B

$$f(x) = \frac{\frac{2}{3}}{4}x^4 - \left(\frac{\frac{1}{2}}{-2}x^{-2}\right) + 5x + c$$

$$= \frac{8}{3}x^4 + x^{-2} + 5x + c.$$

C

$$\int \frac{2}{3}x^3 - \frac{1}{2x^3} + 5$$

$$= \frac{1}{6}x^4 - \frac{1}{8}x^4 + 5x + c.$$

$$= \cancel{\frac{2}{24}x} \ \frac{1}{24}x^4 + 5x + c.$$

4.

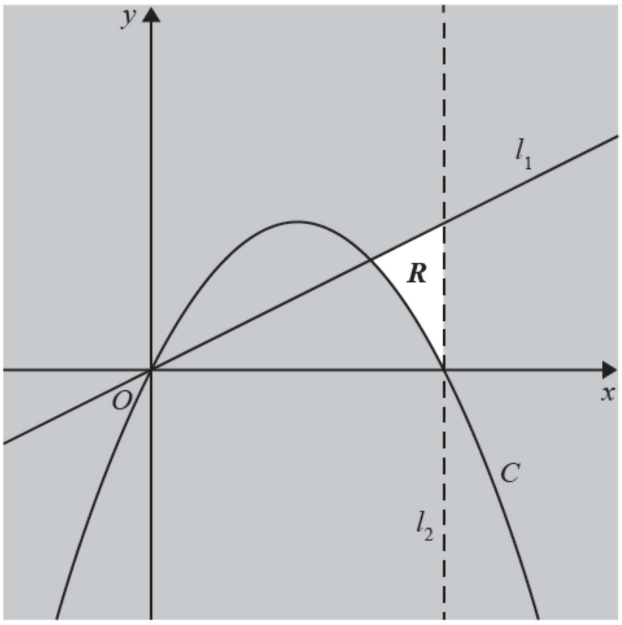

Figure 1

Figure 1 shows a line l_1 with equation $2y = x$ and a curve C with equation $y = 2x - \frac{1}{8}x^2$

The region R, shown unshaded in Figure 1, is bounded by the line l_1, the curve C and a line l_2

Given that l_2 is parallel to the y-axis and passes through the intercept of C with the positive x-axis, identify the inequalities that define R.

(3)

Pearson

Question Number	Scheme	Marks
4.	When ---- represents < or > and —— represents $\le$ or $\ge$	
	Either $2y \le x$ or $y \ge 2x-\frac{1}{8}x^2$	B1
	$2x-\frac{1}{8}x^2=0 \Rightarrow x=16 \Rightarrow x<\ldots$ or $x\le\ldots$	M1
	$x<16,\ 2y\le x$ and $y\ge 2x-\frac{1}{8}x^2$	A1
		(3)
		(3 marks)
Alt1	When ---- represents $\le$ or $\ge$ and —— represents < or >	
	Either $2y<x$ or $y>2x-\frac{1}{8}x^2$	B1
	$2x-\frac{1}{8}x^2=0 \Rightarrow x=16 \Rightarrow x<\ldots$ or $x\le\ldots$	M1
	$x\le16,\ 2y<x$ and $y>2x-\frac{1}{8}x^2$	A1
		(3)

B1 Sight of $2y\le x$ or $y\ge 2x-\frac{1}{8}x^2$. Either inequality is sufficient for B1 and they may be written in an equivalent correct form (see NB below)

NB Inequalities cannot be in terms of R

M1 Attempts to find the upper bound for x to define R. Solves to find where the quadratic intersects the x-axis and then uses their value to write $x<\ldots$ or $x\le\ldots$. Use general principles for solving a quadratic equation (page 5). They do not need to find or state $x=0$ and ignore any lower bound eg $0<x<\ldots$

A1 $2y\le x,\ \ y\ge 2x-\frac{1}{8}x^2$ and $x<16$ (Allow $A\le x<16$ where $A\le 12$).

Candidates may write more than one inequality for a particular boundary. In these cases mark the last one. Correct inequalities labelled on the graph are also acceptable, however, an inequality written below takes precedence.

NB You may see $y\leqslant\frac{x}{2}$ for $2y\leqslant x$ or even $2x-\frac{1}{8}x^2\le y\le\frac{x}{2}$ oe

Alternatively, some candidates may express their inequalities involving a boundary for a dashed line using $\le$ or $\ge$ and a boundary for a solid line using < or >. It may not always be clear so mark positively. See Alt1

Pearson

D

ans : $R < \frac{x}{2}$

$R < x + c$

$R > 2x - \frac{1}{8}x^2$

E

l_2 when $y=0$ $x=$

$2x - \frac{1}{8}x^2 = 0$.

$x(2 - \frac{1}{8}x) = 0$.

$\frac{1}{8}x = 2$

$x = 16$

$x < 16$

$R =$ $2y \leq x$

$y \leq 2x - \frac{1}{8}x^2$.

$x < 16$

F

$0 = 2x - \frac{1}{8}x^2$

$x(2 - \frac{1}{8}x)$

$2 - \frac{1}{8}x = 0$

$\cancel{\frac{-1}{8}} + \cancel{-\frac{1}{8}}x = -2 \cancel{+\frac{1}{8}}$

$x = 0$,

$x = \frac{1}{4}$

$\cancel{(\frac{1}{4}, 0)}$

$L_1 = \frac{2y}{2} = \frac{x}{2}$

$y = \frac{1}{2}x$

$y \geq 2x - \frac{1}{8}x^2$

$y \leq \frac{1}{2}x$

$y < \frac{1}{4}$